CCN2241 Discrete Structures

Assignment 1 (Due 5:00pm, 16 October 2018)

Instructions: This is an individual take-home assignment. Answer ALL questions. You may either type your answers using word processor or hand-write them *very* neatly. Please submit your assignment to your lecturer's pigeon-hole, as follows:

Classes 101 and 103 (KC Wong): HHB **15/F #026**

Class 102 (Joe Lin): HHB **16/F #199**

Question 1 (18 marks)

(a) Write a truth table for the proposition $(r \rightarrow p \vee \neg q) \oplus (\neg r \wedge q \leftrightarrow \neg p)$. (8 marks)

(b) Use a truth table to determine whether the following argument is valid.

1. $(p \wedge q) \leftrightarrow r$
2. q
3. $s \vee r$

$/\therefore \neg (p \wedge \neg r)$ (10 marks)

Question 2 (18 marks)

(a) Use equivalence laws and inference rules to show that the following argument is valid.

1. $p \rightarrow (q \wedge r)$
2. $(q \vee s) \rightarrow p$

$/\therefore p \leftrightarrow q$ (9 marks)

(b) Use only resolution to show that the following argument is valid.

1. $(p \wedge q) \vee \neg(r \vee s)$
2. $(r \wedge p) \rightarrow (q \rightarrow \neg t)$
3. $u \rightarrow \neg u$
4. $\neg(t \vee u) \rightarrow \neg q$

$/\therefore \neg r$ (9 marks)

Question 3 (18 marks)

(a) Prove by contraposition that if $x^2 - 3x + 2 < 0$ then $1 < x < 2$. (6 marks)

(b) Prove by contradiction that for all integers x and y, if xy is odd, then x and y are both odd. (6 marks)

(c) Prove by cases that $n^3 - n$ is a multiple of 3 for all $n \in \mathbf{N}$. (6 marks)

Question 4 (18 marks)

(a) Determine whether each of the following statements is true or false. **Note that in this question R is the second last digit of your student ID (e.g., if your student ID is 17023586A, then your R = 8)**, and A is a nonempty set. Note that we define A^0 to be the empty set $\varnothing$.

i. $\{a,b\} \in \{a,b,c,\{a,b\},R\}$

ii. $\{a,b\} \subseteq \{a,b,\{a,b\},R\}$

iii. $\{a,b\} \subseteq P(\{a,b,\{a,b\},R\})$

iv. $\{\{a,b\}\} \in P(\{a,b,\{a,b\},R\})$

v. $\{a,b,\{a,b\}\} - \{a,b\} = \{a,b\}$

vi. $A^{R+3} = A^{R+1} \times A^2$

vii. $\varnothing \in \varnothing \times A^R$

viii. $\varnothing = \varnothing \times \varnothing^R$

(8 marks)

(b) For sets A, B, and C, by showing twice one side being a subset of the other side prove that A – (B ∩ C) = (A – B) ∪ (A – C).

(10 marks)

Question 5 (18 marks)

(a)

$$f(x) = \begin{cases} x+1, & x < 0 \\ 2x+b, & x \geq 0 \end{cases}$$

Given the above function $f(x)$ defined on the set of real numbers, determine whether it is 1-1 and whether it is onto for each of the following values of b. (Just circle your answers; no need to prove.)

	Is $f(x)$ 1-1?	Is $f(x)$ onto?
Case $b = 2$	Yes / No	Yes / No
Case $b = 1$	Yes / No	Yes / No
Case $b = 0$	Yes / No	Yes / No

(6 marks)

(b) Prove that the following function $g(x)$ defined on the set of real numbers is a bijection.

$$g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

(12 marks)

Question 6 (10 marks)

Use mathematical induction to prove that $3 + 6 + 12 + \ldots + 3 \cdot 2^{n-1} = 3(2^n - 1)$ for all positive integers n.

(10 marks)

~~ End of Assignment ~~