CCN2241 Discrete Structure
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Question 1

a) S0 = {}

S1 = { a }

S2 = { a, b }

S3 = { a, b, c }

S4 = { a, b, c, d}

S5 = { a, b, c, d, e}

S6 = { a, b, c, d, e, f}

S7 = { a, b, c, d, e, f, g}

S8 = { a, b, c, d, e, f, g, h}

b) aefgh

c) 7

Question 2

a)

U1	U2
V1	X2
W1	Y2
X1	V2
Y1	W2
Z1	Z2

	U1	V1	W1	X1	Y1	Z1
U1	0	1	0	1	0	0
V1	1	0	1	0	1	0
W1	0	1	0	0	0	1
X1	1	0	0	0	1	0
Y1	0	1	0	1	0	1
Z1	0	0	1	0	1	0

	U2	X2	Y2	V2	W2	Z2
U2	0	1	0	1	0	0
X2	1	0	1	0	1	0
Y2	0	1	0	0	0	1
V2	1	0	0	0	1	0
W2	0	1	0	1	0	1
Z2	0	0	1	0	1	0

b)

$$\begin{matrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{matrix} \qquad \begin{matrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{matrix}$$

Question 3

ai) By Prim's Algorithm,

Choice	Edge	Weight
1	AC	3
2	CD	4
3	DH	2
4	GH	1
5	EH	2
6	GF	2
7	FI	3
8	AB	4

aii) By Kruskal's Algorithm,

Choice	Edge	Weight
1	AC	3
2	GH	1
3	DH	2
4	EH	2
5	FG	2
6	FI	3
7	CD	4
8	AB	4

b) 21

Question 4

	L1	L2	L3	L4	L5	L6	L7	L8	L9
L1	0	0	0	1	0	0	1	1	1
L2	0	0	0	1	1	1	0	1	0
L3	0	0	0	0	0	1	0	0	1
L4	1	1	0	0	0	1	1	1	0
L5	0	1	0	0	0	0	0	1	0
L6	0	1	1	1	0	0	0	1	1
L7	1	0	0	1	0	0	0	0	0
L8	1	1	0	1	1	1	0	0	0
L9	1	1	1	0	0	1	0	0	0

The problem can be solve be using graph model, with vertices representing the traffic directions, with an edge if there is a collision between two directions.

As the graph cannot form bipartite graph, that is, contains at least one intersection between edges. the chromatic number is > 2.

Because the chromatic number of this graph is 4, 4 phrases is needed.

Phrases	Directions
1	L1, L5, L6
2	L2, L7
3	L3, L8
4	L4, L9

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Question 5

Lemma 1: Every planar graph contains a vertex with deg(v)≤5 .

Base case, P(n≤5) **:** Since there exist ≤5 nodes in G, the graph can be colored using 5 colors.
Inductive step, P(n+1): Assuming P(n) is true, that is, every planar graph with n vertices, we need to show P(n) is true.

By Lemma 1, we know every planar graph has one vertex with deg(v)≤5. We call this vertex v in our graph G. Remove v and for the remaining subgraph G′ we can assume P(n).

If deg(v)≤4, we can color all vertices adjacent to v using 4 colors and use color 5 to color v itself to reach a valid coloring.

If deg(v)=5, we assume that all vertices adjacent to v are colored in different colors.

Assume there exists no path from A to C. We can change the color of A from red to blue, the color of C. Since no neighbor of v now has the color red, we can color v in red. We also need to change all vertices adjacent to AA from blue to red. Since no path exists from AA to CC, the color of CC remains unchanged.

Assume there exists a path from A to C, alernating in color from red to blue. Note that this path bounds a planar embedding with B on the inside and D on the outside. We can change the color of B from green to yellow, the color of D. Since no neighbor of v now has the color green, we can color v in green. We also need to change all vertices adjacent to B from yellow to green. Since no path can exist from B to D (without crossing the path from A to C), the color of D remains unchanged.